

**Author: Jan Sinopoli**

**Prompt:**

A group of high school mathematics teachers attended a training session regarding a new algebra one curriculum. One lesson focused on writing equations for given situations, substituting a value for either the independent variable or dependent variable, and solving for the unknown. A particular situation involved the fact that  $\frac{2}{5}$  of the students attending a university were women. Introductory questions asked students to write ratios and/or fractions regarding the number of women, men and total students prior to writing an equation relating the total number of students and the number of women attending the university. A discussion about fractions, ratios and rates revealed that teachers had some confusion regarding these terms.

**Mathematical Foci**

**Mathematical Focus 1:**

Conceptual Definitions:

A fraction is a part to whole relationship.

A ratio is a part to part or part to whole relationship.  
The quantities may or may not have the same unit.

A rate is a ratio statement with quantities of different units.

**Mathematical Focus 2:**

Notation:

Fractional notation is used to represent fraction, ratio and rate relationships.  
Fractional notation is used to express quotients.

**Mathematical Focus 3:**

Slope is referred to as a rate, rise/run. A constant rate signifies that a function is linear.

### **Prompt:**

A teacher introduced a task she wrote involving quadratic functions to her colleagues. The task involved writing the equation for vertical motion for a particular situation, using  $v = -16t^2 + v_0 t + h_0$ , where  $t = \text{time (second)s}$ ,  $h = \text{initial height(feet)}$ , and  $v = \text{velocity(feet per second)}$ . The task asked for an equation to be written given the initial velocity and height. The equation was to be graphed (time vs. height), then there were questions to determine the height for a given time, or the time(s) for a given height. After solving the task, one teacher asked, "Why is this task called 'The Path of a Ball'? The graph doesn't represent its path, does it?"

### **Mathematical Foci**

#### **Mathematical Focus 1:**

Vertical motion problems and the formula  $v = -16t^2 + v_0 t + h_0$ , where  $t = \text{time (second)s}$ ,  $h = \text{initial height(feet)}$ , and  $v = \text{velocity(feet per second)}$  track data regarding time and height only. The graph documents the height for any given time. Vertical motion problems involve an object being launched vertically, so the horizontal distance remains the same throughout the entire problem.

#### **Mathematical Focus 2:**

A graph that represents the path of a ball being thrown would have the horizontal distance represented along the x-axis and the vertical distance represented along the y-axis. Time would be used to calculate the path of the ball, but the graph would only document the position of the ball.

To construct an equation and graph to represent the path of a ball, it is necessary to know the angle of elevation of the ball when it is being thrown to account for vertical and horizontal motion, as well as the initial height and velocity.

The path of a ball can be modeled via parametric equations:

$$\text{Horizontal: } (V_0 \cos \theta) t$$

$$\text{Vertical: } -16 t^2 + V_0 (\sin \theta) t + h_0$$

(where  $V_0$  is initial velocity and  $h_0$  is initial height, units are in feet and seconds)